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Thermodynamic analysis of convective heat transfer in a packed duct with asymmetrical wall temperatures

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Abstract—Combination of the first and second law of thermodynamics has been utilized in analysing the convective heat transfer in a rectangular packed duct. Raschig ring type of packing in the air flow passage is used to enhance the heat transfer from uniformly heated front wall to air when the other walls are adiabatic. The packing increases wall to fluid heat transfer considerably, hence reduce the entropy generation due to the heat transfer across a finite temperature difference. However, the entropy generation due to fluid flow friction increases. The net entropy generations resulting from the above effects provide a new criterion in analysing the system. Using the Ergun equation for pressure drop estimation, an expression for the volumetric rate of entropy generation has been derived for a vertical 0.675 m long packed duct ($H/W = 0.31$). This expression has been displayed graphically to show the influences of physical and geometric parameters on the entropy generation. Introduction of packing proves to be a thermodynamically sound augmentation technique of convective heat transfer. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

Thermodynamic analysis has recently been a topic of wide interest in the field of thermal design and heat transfer, such as power plants, heat exchangers, energy storage and electronic cooling devices. Since the convection heat transfer rate in a smooth channel is normally insufficient for real applications, the augmentation of heat transfer becomes of necessity. Entropy generation in heat transfer augmentation techniques are widely explored [1–4]. Recently, the following three augmentation techniques for convective heat transfer in a channel have been analysed: introduction of wire-coil inserts [5, 6], and packing [7] into channel with constant wall temperatures, and transverse fin array [8] in a duct with asymmetrical wall temperatures which is one important application and design concern. For such systems there are various combinations of thermal boundary conditions that are detailed by Gao and Hartnett [9].

In an early study, Colburn [10] reported that the rate of forced convection heat transfer from uniformly heated wall to air through a packed tube is about eight times higher than that of an empty tube. Experimental investigation of heat transfer in a rectangular packed duct with asymmetrical wall temperatures has also shown about two-fold [11], and three-fold [12] increases in the wall fluid heat transfer coefficient: this augmentation technique is readily applied to solar air heating systems to increase the thermal efficiency [13, 14]. The packing causes mixing and prevents the build-up of a slow moving thin layer of fluid next to the

wall and therefore increases the radial transfer of heat within the fluid. However, packing also increases frictional losses hence pumping power. Clearly, while attempting to upgrade the thermal performance of the duct one runs the risk of defeating the purpose of enhancement of heat transfer. The combined utilization of the first and second laws of thermodynamics yields a new approach to analyse such a system based on the net entropy generation (irreversibility) [1, 3].

Previously an experimental study of convective heat transfer in laminar flow with Re 250–750, was carried out in a 0.675 m long vertical rectangular duct ($H/W = 0.31$) packed with Raschig rings in four different sizes. Front wall of the duct was supplied by a constant heat flux, while the back and side walls are insulated [11]. For this system the Ergun equation has been introduced into the combination of the first and second law of thermodynamics and the volumetric rate of entropy generation has been evaluated analytically and displayed graphically showing the results of physical and geometric parameters.

ENTROPY GENERATION

The non-equilibrium phenomenon of exchange and momentum within the fluid and at the solid boundaries causes of continuous generation of entropy in the flow field. Local entropy generations per unit volume S''' of an incompressible Newtonian fluid for a two-dimensional channel are represented by [3, 15]:

NOMENCLATURE

a_p	specific surface [$\text{m}^2 \text{m}^{-3}$]	u	velocity [m s^{-1}]
A	parameter given by equation (10)	W	width of duct [m]
c_p	specific heat at constant temperature [$\text{J kg}^{-1} \text{K}^{-1}$]	x	direction of fluid flow
D	D_p/H	X	x/H
D_e	equivalent diameter of duct [m]	y	direction normal to the flow direction.
D_p	equivalent diameter of packing [m]	Greek symbols	
G	mass velocity [$\text{kg m}^{-2} \text{s}^{-1}$]	α	aspect ratio (H/W)
H	depth of duct [m]	α_{ef}	effective thermal diffusivity [$\text{m}^2 \text{s}^{-1}$]
J	$S''_{gen} (k_e T_0^2 / Q^2)$ dimensionless	ε	void fraction
k_f	thermal conductivity of fluid [$\text{W m}^{-1} \text{K}^{-1}$]	μ	Newtonian fluid viscosity [$\text{kg m}^{-1} \text{s}^{-1}$]
k_{ef}	effective thermal conductivity of fluid [$\text{W m}^{-1} \text{K}^{-1}$]	ρ	density [kg m^{-3}]
K	parameter defined in equation (8)	ϕ	ratio of entropy generation by friction to that of heat transfer.
L	flow path length [m]	Subscripts	
Nu	Nusselt number, ($Nu = hD_e/k_f$)	b	bulk
Q	heat flux rate [W m^{-2}]	calc	calculated
Re_p	Reynolds number, ($Re = (GD_p/\mu)$)	ef	effective
Re	Reynolds number, ($Re = (GD_{eq}/\mu)$)	e	equivalent
Ref	product of Re and friction coefficient	expt	experimental
S''	cross-sectional entropy generation	f	fluid
S'''	volumetric rate of entropy generation [$\text{W m}^{-3} \text{K}^{-1}$]	p	packing
St	Stanton number, ($St = (h/\rho u_b c_p)$)	w	wall
T	temperature [K]	ΔP	finite pressure
ΔT_b	$T_b - T_0$	ΔT	finite temperature.

$$S''' = \frac{k}{T^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T} \left\{ 2 \left(\frac{\partial u_x}{\partial x} \right)^2 + 2 \left(\frac{\partial u_y}{\partial y} \right)^2 + \left[\left(\frac{\partial u_x}{\partial y} \right) + \left(\frac{\partial u_y}{\partial x} \right) \right]^2 \right\} \quad (1)$$

showing the entropy generation due to finite differences in the x - and y -directions, and due to the fluid friction by the first and second term, respectively. Entropy generation profiles may be constructed using equation (1) if the velocity and the temperature fields are known in the heat transfer medium.

The duct under consideration is shown in Fig. 1. It is assumed that wall-to-fluid bulk temperature is small enough, and there is no considerable change in physical properties of the fluid, no axial conduction, and no natural convection. Slug flow conditions ($u = u_b$) with the property of usual maximum near the wall followed by the lower, and essentially constant velocity right up to the center [16] prevail through the cross-section of the packed flow passage. The velocity may be related to the pressure by inviscid-flow behavior ($dP/\rho = -d(u_b^2/2)$), and using the Bernoulli equation free stream velocity gradient in flow direction is expressed by [17]:

$$\frac{du}{dx} = \frac{du_b}{dx} = \frac{1}{\rho u_b} \left(-\frac{dP}{dx} \right) \quad (2)$$

The pressure gradient ($-dP/dx$) can be evaluated from the Ergun equation [18]:

$$-\frac{dP}{dx} = \frac{1}{\rho} \left[\left(\frac{C_1(1-\varepsilon)^2}{\varepsilon^3 D_p^2} \mu \right) + \left(\frac{C_2(1-\varepsilon)}{\varepsilon^3 D_p} \right) G \right] G \quad (3)$$

where the first term represents the viscous and the second term shows the inertial resistances for fluid flow. The constant C_1 was reported as 130 and C_2 was given by [19]:

$$C_2 = \frac{D_e/D_p}{0.335(D_e/D_p) + 2.28} \quad (4)$$

The equivalent diameter of the duct D_e and the packing D_p may be obtained from the following relations [20]:

$$D_e = \left(3.05 \frac{H^2 W^2}{(H+W)} \right)^{1/3} \quad (5)$$

$$D_p = \frac{6(1-\varepsilon)}{0.3a_p} \quad (6)$$

With the void fraction ε expressed in terms of the ratio of packing diameter to separation distance between plates $D = D_p/H$ and aspect ratio α as $\varepsilon = [1 + 0.33(1 + \alpha)D]^{-1}$, and after substituting equa-

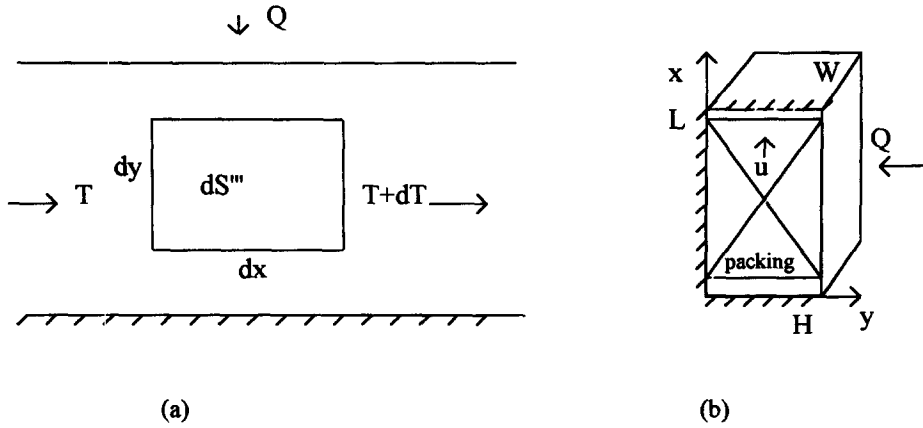


Fig. 1. (a) Control volume of the packed duct; (b) schematic view of the packed duct.

tion (3) into equation (2) the expression for (du/dx) reduces to:

$$\frac{du}{dx} = Ku_b, \quad (7)$$

where:

$$K = \left[\left(\frac{0.33(1+\alpha)D}{1+0.33(1+\alpha)D} \left(\frac{C_1}{Re_p} \right) + C_2 \right) \times \frac{0.33(1+\alpha)}{H} [1+0.33(1+\alpha)D^2] \right]. \quad (8)$$

Solving the energy balance for fully developed temperature field:

$$\frac{\partial}{\partial y} \left(\frac{\partial T}{\partial y} \right) = \frac{u}{\alpha_{ef}} \left(\frac{dT_b}{dx} \right) \quad (9)$$

with the boundary conditions of insulated wall and heated wall by constant heat flux Q :

$$\text{at } y = 0 \quad \partial T / \partial y = 0,$$

$$\text{at } y = H \quad -k_{ef}(\partial T / \partial y) = Q = \text{constant}$$

temperature profile is expressed by:

$$T = T_0(1 + \tau A) \quad (10)$$

where:

$$A = \frac{hH}{2k_{ef}}(Y^2 - 1) + StX + 1$$

and

$$\tau = \frac{Q/h}{T_0} = \frac{T_w - T_b}{T_0} \quad Y = \frac{y}{H} \quad X = \frac{x}{H}.$$

The effective thermal conductivity k_{ef} in equation (10) may be expressed by:

$$K_{ef} = k_f \left[\varepsilon + (1-\varepsilon) \left(0.2 + \frac{2k_f}{3k_p} \right)^{-1} \right] + Re_p \left(\frac{0.0025}{1+46D^2} \right) \quad (11)$$

which combines the static and dynamic contributions, shown by the first and second terms, respectively [21]. The bulk temperature may be expressed by:

$$T_b = \frac{\int_0^1 uTW dY}{\int_0^1 uW dY}. \quad (12)$$

The terms (dT/dx) and u_b may be calculated from the simple energy balance:

$$Q dx = \rho u_b H c_p dT \quad (13)$$

as

$$\frac{\partial T}{\partial x} = \frac{dT_b}{dx} = \frac{Q}{h} \frac{St}{H} \quad (14)$$

and

$$u_b = \frac{QX}{\rho c_p \Delta T_b}. \quad (15)$$

Temperature gradient in the y direction may be obtained from equation (10) and given by:

$$\frac{\partial T}{\partial y} = \frac{Q}{k_{ef}} Y. \quad (16)$$

An expression for the volumetric entropy generation for the packed duct flow under consideration may be reduced from equation (1):

$$S''' = \frac{k_{ef}}{T^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{2\mu}{T} \left(\frac{\partial u}{\partial x} \right)^2. \quad (17)$$

Here the first term shows the entropy generated due to heat transfer $S'''_{\Delta T}$, while the entropy generated due to fluid friction $S'''_{\Delta P}$ is shown by the second term, and the ratio of these is ϕ :

$$\phi = S'''_{\Delta P} / S'''_{\Delta T}. \quad (18)$$

By inserting equations (7), (14) and (16) into equation (17) the following dimensionless expression for

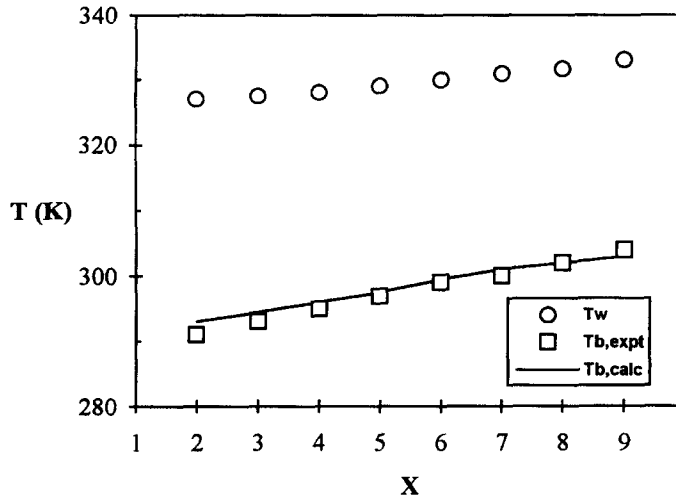


Fig. 2. Air flow and wall temperatures for the Raschig ring hard plastic packed duct.

the volumetric entropy generation number J can be obtained as :

$$J = S''' \frac{k_{ef} T_0^2}{Q^2} = \left(\frac{k_{ef}}{(1 + \tau A)} \right)^2 \left[\left(\frac{St}{hH} \right)^2 + \left(\frac{Y}{k_{ef}} \right)^2 \right] + \frac{2k_{ef} \mu T_0}{(1 + \tau A)} \left(\frac{KX}{\rho c_p \Delta T_b} \right)^2 \quad (19)$$

In equations (19) local entropy generation has been expressed in terms of τ , Y , X and D including the properties of the fluid ρ and c_p .

The rate of dimensionless entropy generation over the cross-section J may be calculated by integration :

$$S'' = \int_0^1 J dY \quad (20)$$

In order to determine the total entropy generation in the duct, equation (20) must be integrated over the entire channel length.

RESULTS AND DISCUSSION

Earlier an experimental study was performed using a vertical rectangular flow passage of the 0.675 m long duct filled with Raschig ring type of packing that was especially preferred to produce flow pressure drop along the flow passage. The duct had a front wall made of 1 mm thick stainless steel, which was heated radiantly with an adjustable electric power input through a resistance wire with a pitch of about 18 mm held 25 mm from the plate surface, while the other walls were insulated. The width W and depth H of the duct were 0.225 m and 0.08 m, respectively giving an aspect ratio of 0.31. The packed bed height in the duct was 0.65 m, and the D_p/D_e changed in the region of 0.0897–0.1020 [11]. Reynolds numbers based on the equivalent diameter of the duct changed between 250 and 750 and Prandtl number was assumed as 0.7. Experimental procedure was detailed in ref. [11].

Figure 2 shows the temperature profile in the axial

direction for the wall and air flow. Using the net increase in the enthalpy of air, Nusselt numbers were determined with changing Reynolds numbers and packing size. It was found that heat transfer coefficients increased about twice after introducing packing into the air flow passage, and the increase produced a peak value at certain packing size [11].

In estimations Y is changed between 0.01 and 0.99, while X varied in the region of 0.05–8.5 for the packed and 5.0–8.5 for the empty duct as an attempt to lessen the entrance and the wall effects, and a single size of hard plastic (polyvinyl chloride) packing is considered. The estimated bulk temperature profile is presented in Figs. 2 and 3 that indicate the satisfactory representation of temperature; the rate of change is rapid for $Y > 0.5$. Figure 4 shows the map of J for $\varepsilon = 0.84$, $Re = 738$, $\tau = 0.0862$, $T_0 = 290$ K, $Q = 122$ W m⁻², $St = 0.0526$ and $D_p = 0.014$ m. The entropy generation is high in the heated wall region. A gradual decrease of J is observed away from the heated wall. Using a similar procedure the volumetric entropy generation may be obtained for the empty duct from the relation :

$$S''' = \frac{k_r Q^2}{T^2} \left[\left(\frac{St}{hH} \right)^2 + \left(\frac{3Y^2 - 2Y^3}{k_r} \right)^2 \right] + \frac{\mu}{2T} \left(\frac{2HReQX}{D_{eq}^2 \rho c_p \Delta T_b} \right)^2 (1 - 2Y)^2 \quad (21)$$

where temperature profile is obtained for fully developed flow with parabolic velocity field :

$$T = T_0 \{ 1 + \tau [hH(2Y^3 - Y^4 - 1)/2k_r + StX + 1] \} \quad (22)$$

Figure 5 shows distribution of J obtained from the equation (21) for an empty duct. For $Re = 300$, $\tau = 0.1$, $T_0 = 290$ K, $Q = 40$ W m⁻² and $St = 0.036$ a sharp change of entropy between $Y = 0.4$ and 0.8 is observed. Similar observations were also made by

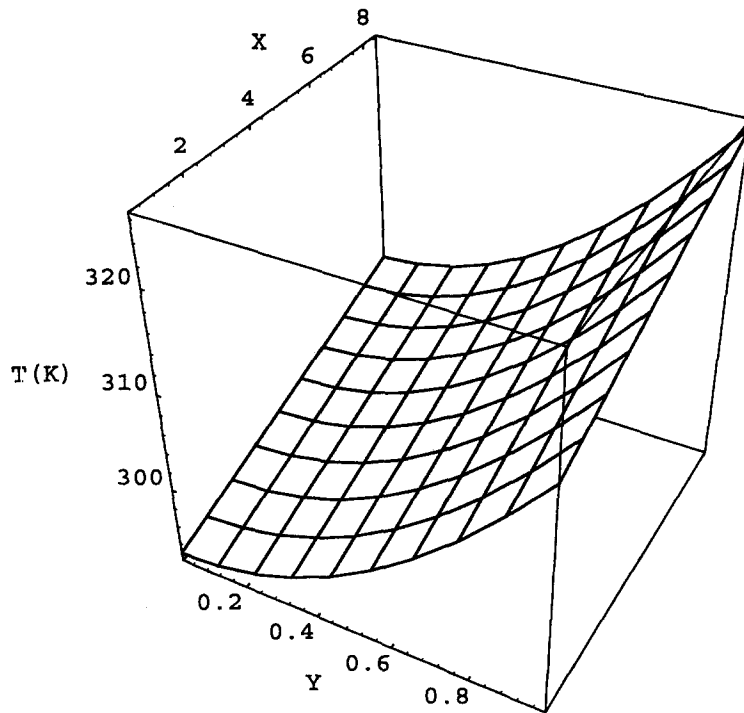


Fig. 3. Temperature profile in the packed duct heated by constant heat flux from one surface while the others are insulated.

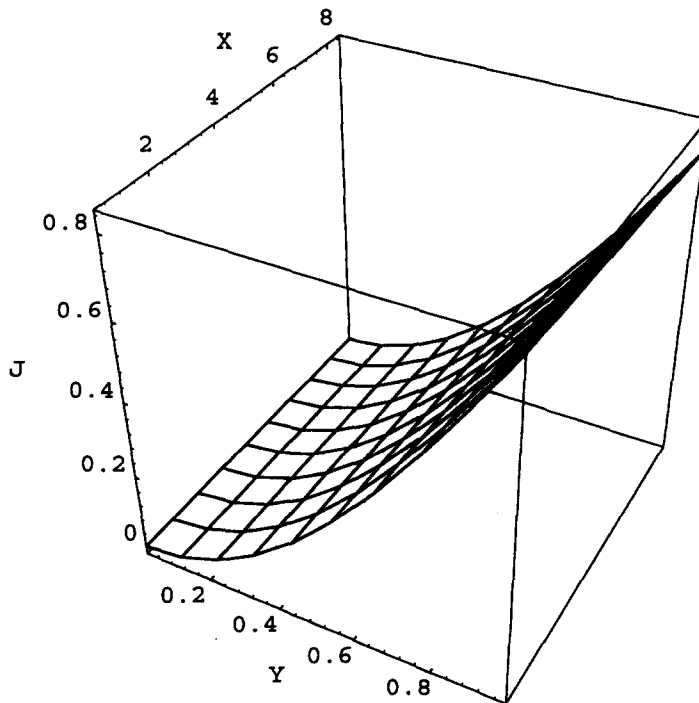


Fig. 4. Non-dimensional entropy generation J profile in the packed bed with asymmetrical wall temperatures.

Bejan [3] and San *et al.* [15]. This distinction of distribution of entropy in the duct is the result of temperature and flow fields occurring with and without packing. Figure 4 indicates equipartition of entropy distribution that leads to a relative minimum of

entropy production [22] in packed duct compared with that of empty duct. Energy and momentum transfer processes are improved, in some economic sense, by distributing the entropy production as evenly as possible along the space variable of the duct [22].

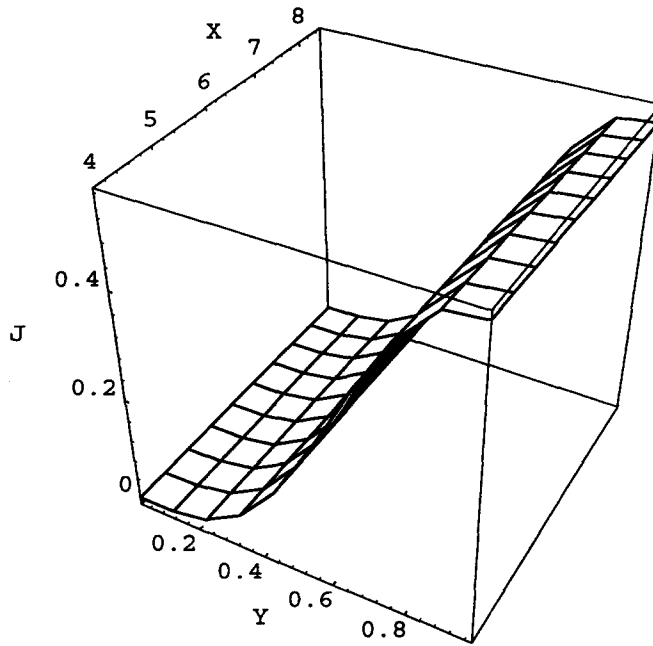


Fig. 5. Non-dimensional entropy generation J profile in the empty duct with asymmetrical wall temperatures.

Figure 6 shows the distribution of ϕ for $\varepsilon = 0.84$, $Re = 738$, $\tau = 0.0862$, $T_0 = 290$ K, $Q = 122$ W m⁻², $St = 0.0526$ and $D_p = 0.014$ m. As seen from this figure, the effect entropy generation due to pressure

drop decreases sharply towards the heated wall from the adiabatic wall and at about $Y = 0.4$, and it is negligible beside the heat transfer effect. For $Re = 525$, $Q = 60$ W m⁻², Figs. 7 and 8 show the

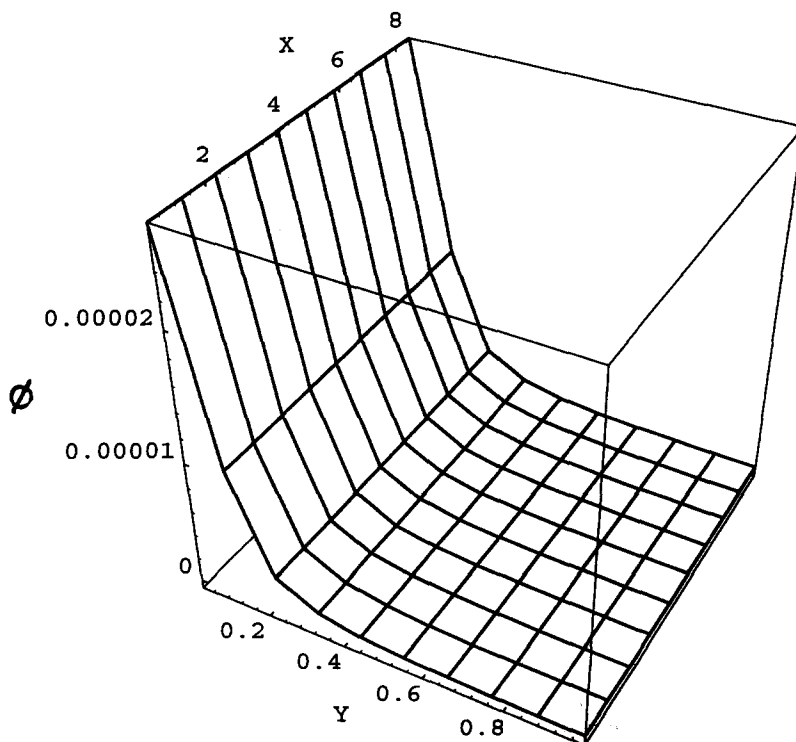


Fig. 6. The distribution of ratio of entropy generation by the friction to that by heat transfer ϕ in the packed duct.

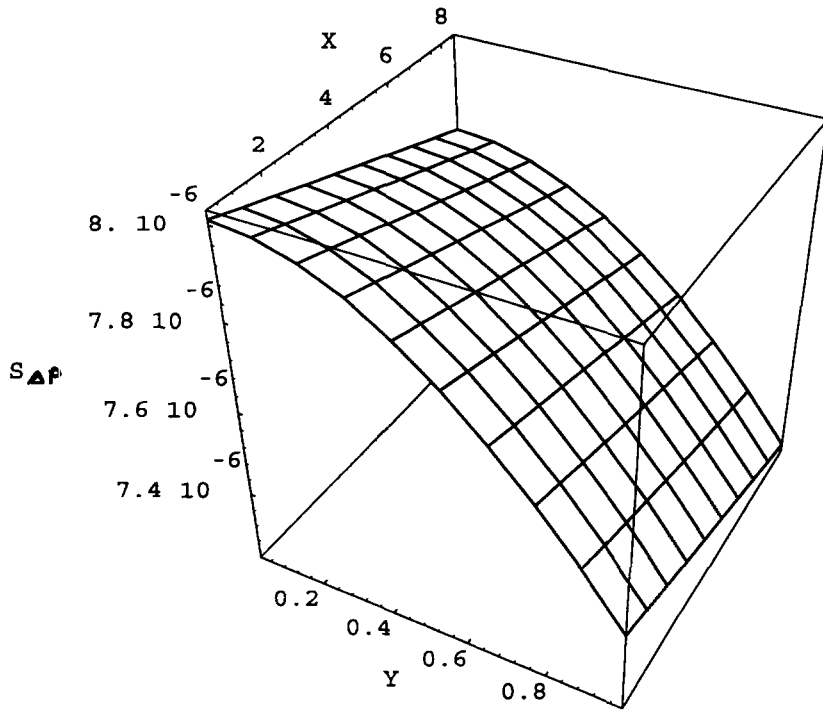


Fig. 7. The distribution of volumetric entropy generation due to pressure drop $S''_{\Delta P}$ in the packed duct.

distribution of $S_{\Delta P}$ over the flow passage in the packed and empty ducts, respectively. The effect of parabolic flow field is clearly seen in Fig. 8 and elevation of entropy due to the pressure drop caused by the packing is marginal.

The cross-sectional entropy generation S'' has been calculated from equation (20) using numerical inte-

gration by the MATHEMATICA. Figure 9 shows the effect of Reynolds number on the variation of S'' in the flow direction of the packed duct for $\epsilon = 0.84$, $\tau = 0.0862$, $T_0 = 290$ K, $Q = 122$ W m⁻², $St = 0.0526$ and $D_p = 0.014$ m. As Re increases the cross-sectional entropy generation shows a gradual decrease. In Fig. 10 the effect of packing equivalent size on the cross-

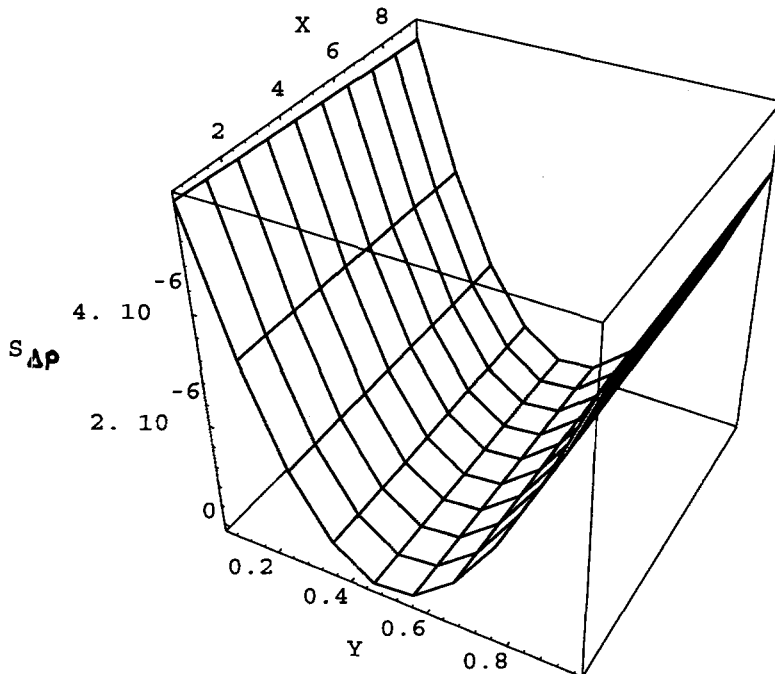


Fig. 8. The distribution of volumetric entropy generation due to pressure drop $S''_{\Delta P}$ in the empty duct.

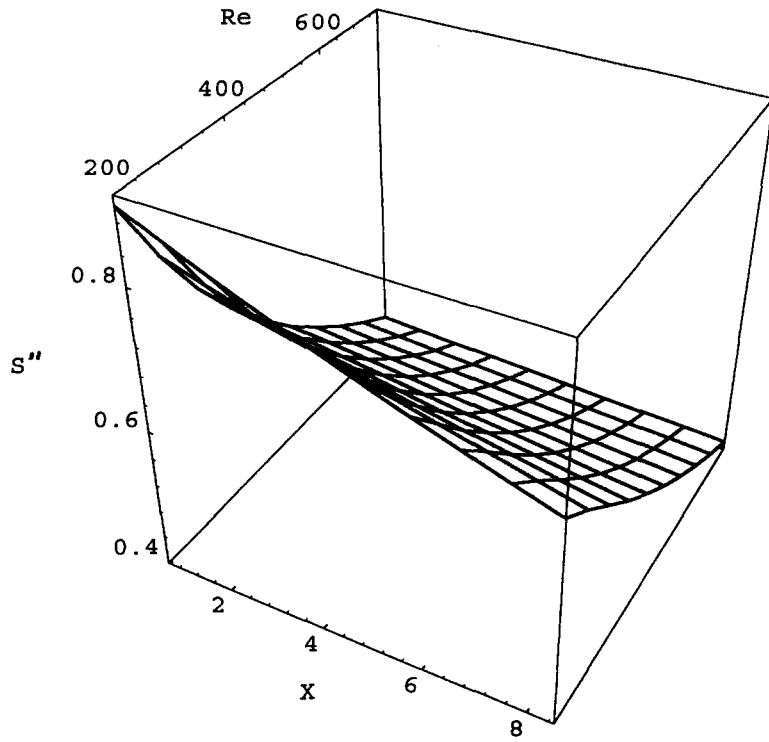


Fig. 9. Influence of Reynolds number on cross-sectional entropy generation S'' in the packed duct.

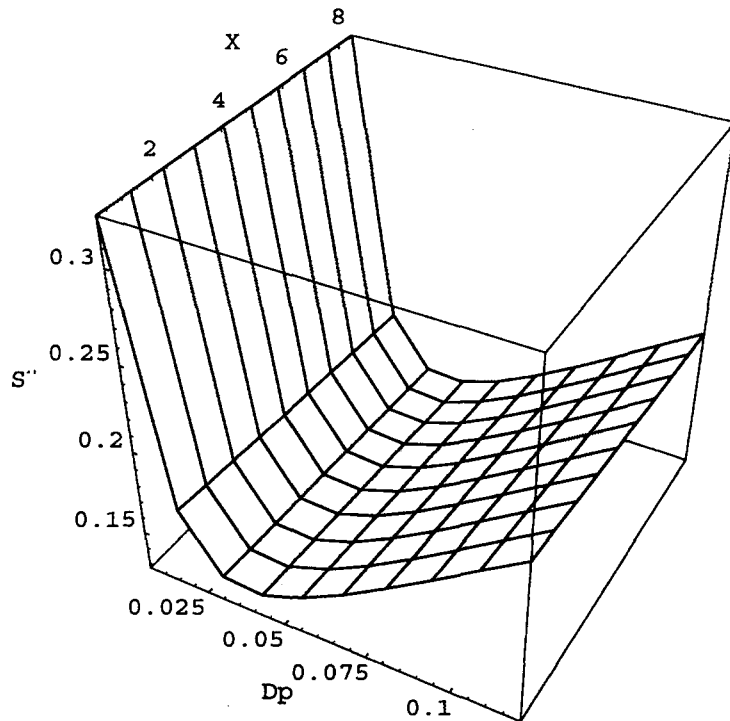


Fig. 10. Effect of packing size on cross-sectional entropy generation S'' in the packed bed.

sectional entropy generation is seen: a minimum value of S'' for certain packing size that is consistent with the reported peak values of increases of heat transfer coefficients [10, 11] for a certain packing size.

CONCLUSIONS

Using the combination of the first and second law of thermodynamics, together with the temperature and velocity gradients, an expression for the volumetric entropy generation in a packed duct with asymmetrical wall temperatures has been derived and displayed graphically. Raschig ring type of packing has been used in the air flow passage underneath the heated wall with constant heat flux. The influences of Re and packing size on cross-sectional entropy generation are evaluated. Entropy generation under the configuration of packed duct is only slightly elevated. For a specified heat transfer duty, the local rate of entropy generation is closer to the configuration of uniformly distributed (equipartitioned) along the space compared with that of empty duct. This implies a relative minimum entropy production [22]. Therefore such a configuration is recommended based on the consideration of thermodynamic analysis.

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